# Masses of $cq\overline{qq}$ Tetraquarks Using Fermi-Breit Hyperfine Interaction

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# Quarks as elementary particles

The idea of the quark model is to invent a small set of particles, assign them appropriate properties, and use them to construct the hadrons much as neutrons and protons are used to construct the various atomic nuclei. The hypothetical fundamental building blocks are called quarks. They have never been isolated and have never been observed.

Quarks: (u, d) (c, s) (t, b) + all their antiparticles

Some quantum numbers of light quarks: *u*, *d*, *s*.

| quark | B | T | $T_3$ | σ | S | Y | Q |
|---|---|---|---|---|---|---|---|
| ***u*** | 1/3 | 1/2 | 1/2 | 1/2 | 0 | 1/3 | 2/3 |
| ***d*** | 1/3 | 1/2 | -1/2 | 1/2 | 0 | 1/3 | -1/3 |
| ***s*** | 1/3 | 0 | 0 | 1/2 | -1 | -2/3 | -1/3 |

- B - baryon number
- T - magnitude of isospin
- $T_3$ - 3-component of isospin
- σ - spin in units of $\hbar$
- Y – hypercharge
- Q - charge in units of *e*
- *S* - strangeness

# SU(3) operators

There is a group of operators (eight in number) which do not change the interaction when they operate on it.

- the SU(3) indicates that the basis of the group consists of 3 independent states (the 3 quarks)
- every operator which operates in a space which is specified by 3 basis states can be written as a linear combination of these 8, augmented by the identity operator

SU(3) provides the foundation for grouping the hadrons into supermultiplets, as the octets, decimets and singlets are collectively called.

# Scalar $cq\overline{qq}$ tetraquarks

- they are composed of a charm quark *c* and of the three light flavors *u*, *d*, *s*
- total spin of this system is 0

$Q = T_3 + Y/2$; $Y = B + S + C$

B =1/3 for quark, -1/3 for antiquark
S = -1 for *s* quark, 1 for *s*-antiquark
C = 1 for *c* quark, -1 for *c*-antiquark

- For tetraquarks with one *c* quark attached to one light quark and two light antiquarks, we have:

$B = 1/3 + 1/3 – 1/3 – 1/3 = 0$; $C = 1$ => $Y = S + 1$

# 27 tetraquark states

- There are 27 different tetraquarks composed of a charm quark *c* and of the three light flavors *u*, *d*, *s*: 11 cryptoexotic (3 $D_s^+$ , 4 $D^+$, 4 $D^0$) and 16 explicit exotic states
- Tetraquarks with charm quantum number C = 1 are grouped by the same properties, into multiplets with the same baryon number, spin and intrinsic parity
- If a particle belongs to a given multiplet, all of its isospin partners (the same isotopic spin magnitude T and different 3-components $T_3$) belong to the same multiplet
- Mass mixing effects (due to the same quantum numbers) are included in all calculations: which mixed state belongs to one multiplet and which to the other is arbitrary at present and, in fact, the physical particle may be some superposition of the two states.

# Fermi-Breit hyperfine interaction (FB HFI)

- The interaction we use is given by the Hamiltonian operator:
  (D. A. Liberman 1977, Phys. Rev. D **16**, 1542)

$$H_{FB} = C\sum_{i<j}\left(\lambda_i^C \lambda_j^C\right)\left(\vec{\sigma}_i\vec{\sigma}_j\right)$$

$\lambda_i$ - Gell-Mann matrices for color SU(3), $\sigma_i$ - the Pauli spin matrices, C – constant (C > 0).

- This schematic flavor-spin interaction between quarks and antiquarks leads to FB HFI contribution to tetraquark masses:

$$m_{\nu,FB} = \left\langle \nu\uparrow \middle| C\sum_{i<j=2}^{4}\frac{\vec{\sigma}_i\vec{\sigma}_j}{m_i m_j}\left(\lambda_i^C\lambda_j^C\right)\middle|\nu\uparrow\right\rangle$$

$m_i$ - the constituent quark effective masses: $m_u = m_d \neq m_s$

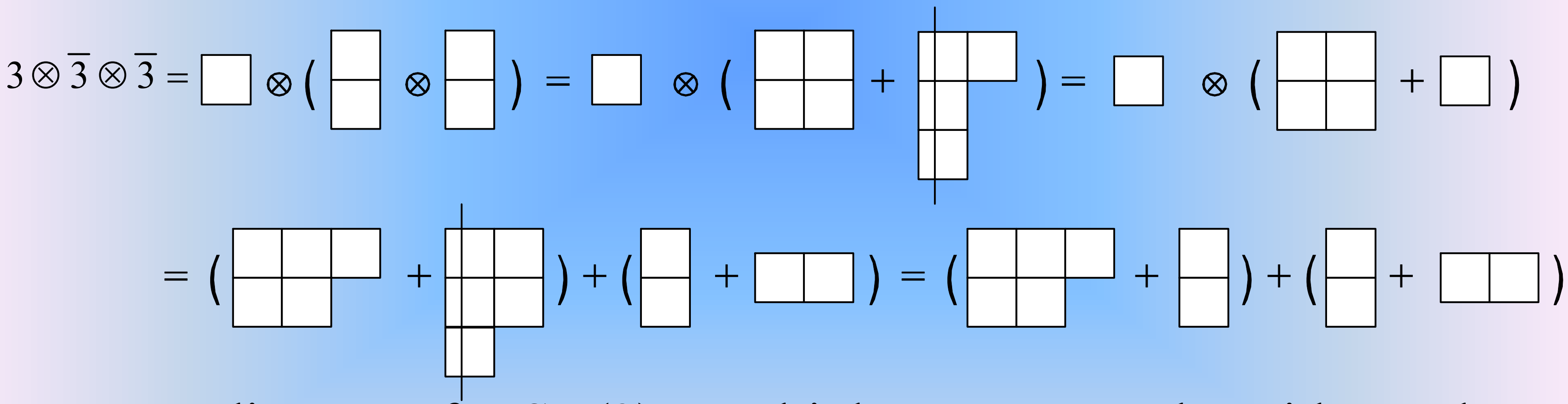
$\nu$ - flavor wave functions

$m_\nu = m_{\nu,0} + m_{\nu,FB}$ total quark masses

# SU(3) flavor multiplets

Under the transformation of $SU(3)_F$, the charm quark is singlet. There are four multiplets according to:

$$3\otimes\bar{3}\otimes\bar{3}\otimes 1=\overline{15}_S+\bar{3}_S+6_A+\bar{3}_A$$



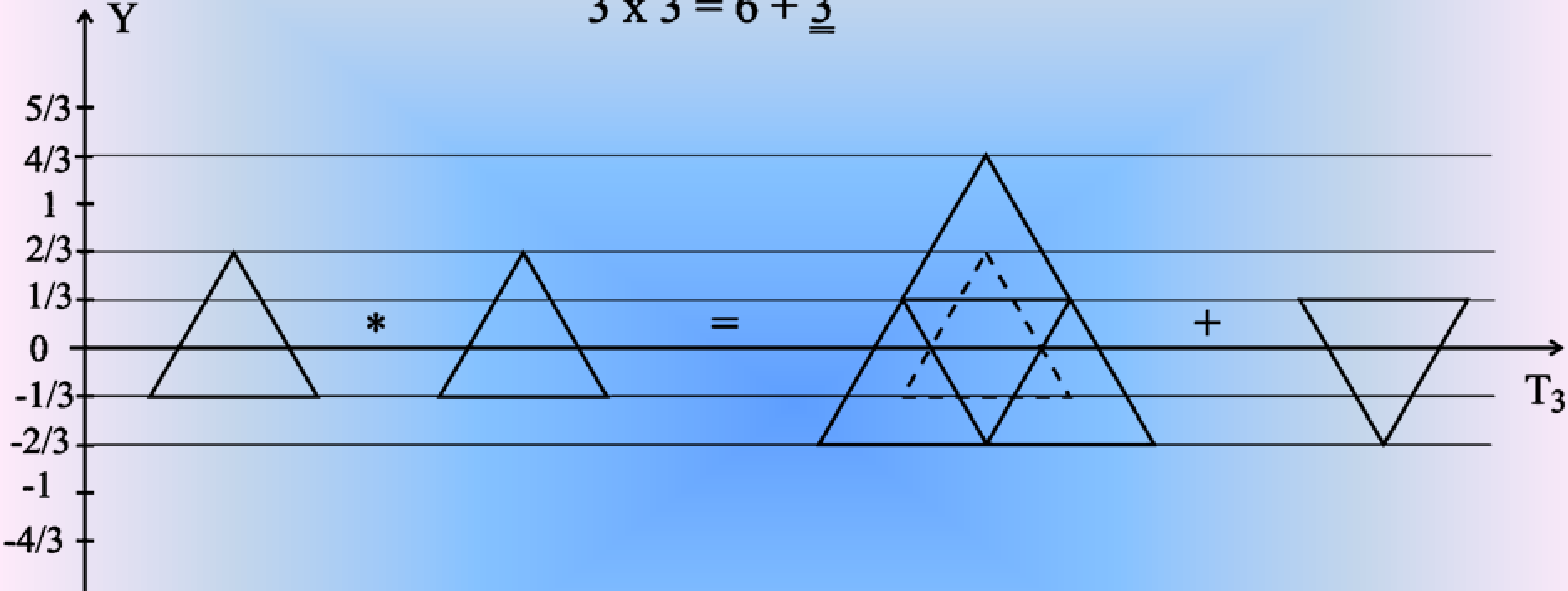

Young diagrams for SU(3)F multiplets. Tetraquarks with quark content $cq\bar{q}\bar{q}$ form four multiplets: two anti-triplets, one anti-15-plet and one sextet.

# Weight diagrams

- We plot the eigenvalues of $T_3$ and Y that occur for the quarks in a representation as points in the $T_3$ - Y plane
- The quantum numbers Y and $T_3$ are additive and thus their values for a $cq\overline{qq}$ state are obtained by simply adding the values for $q$ or $\overline{q}$
- The points of the product of two representations are obtained by taking every point of one diagram to be the origin of another diagram
- The subscripts S and A on the multiplets indicate that the flavor states are symmetric or antisymmetric under interchange of the first two quarks, or under interchange of the last two antiquarks.

# WEIGHT DIAGRAMS FOR $q\bar{q}\bar{q}$ (I)

$$\bar{3} \times \bar{3} = \bar{6} + \underline{\underline{3}}$$

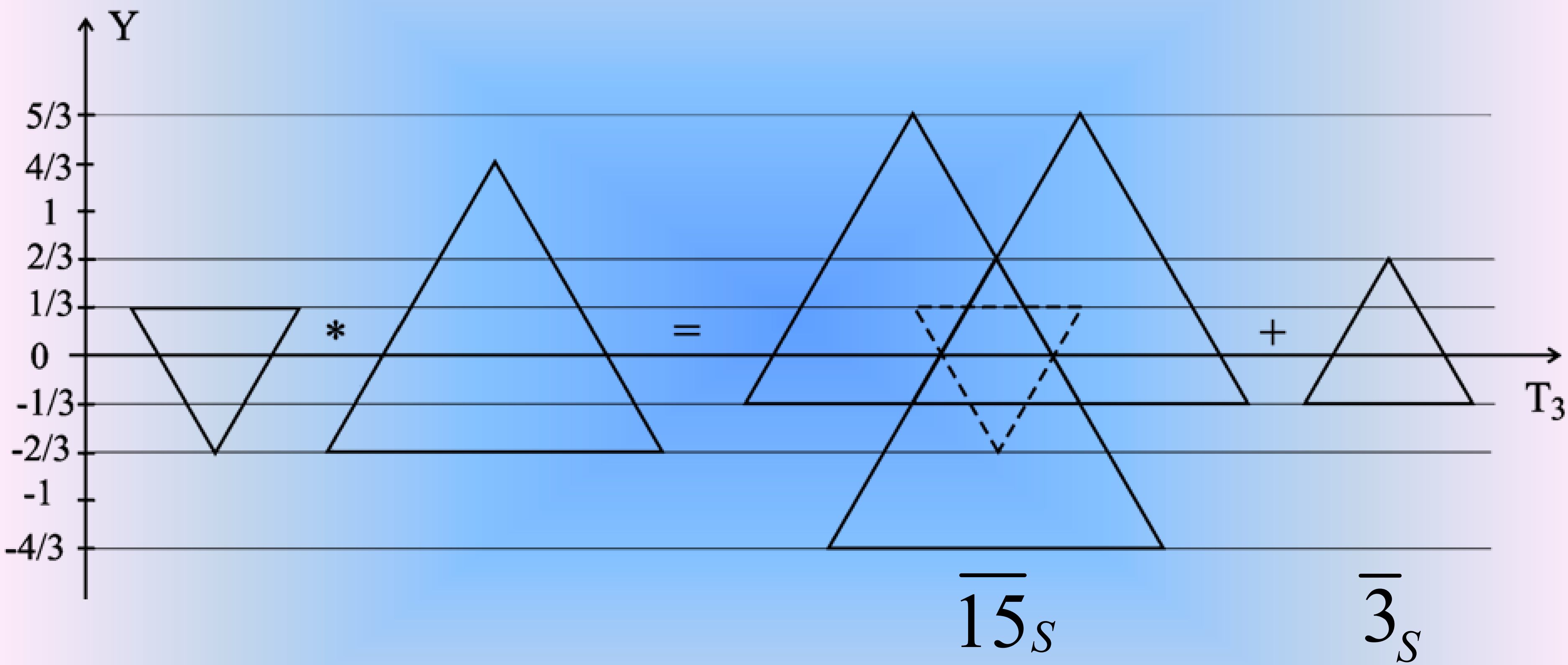



The ordinate shows hypercharge Y and abscissa 3-component $T_3$ of isotopic spin magnitude.

We first combine two of the antiquarks.

## WEIGHT DIAGRAMS FOR $q\bar{q}\bar{q}$ (II)

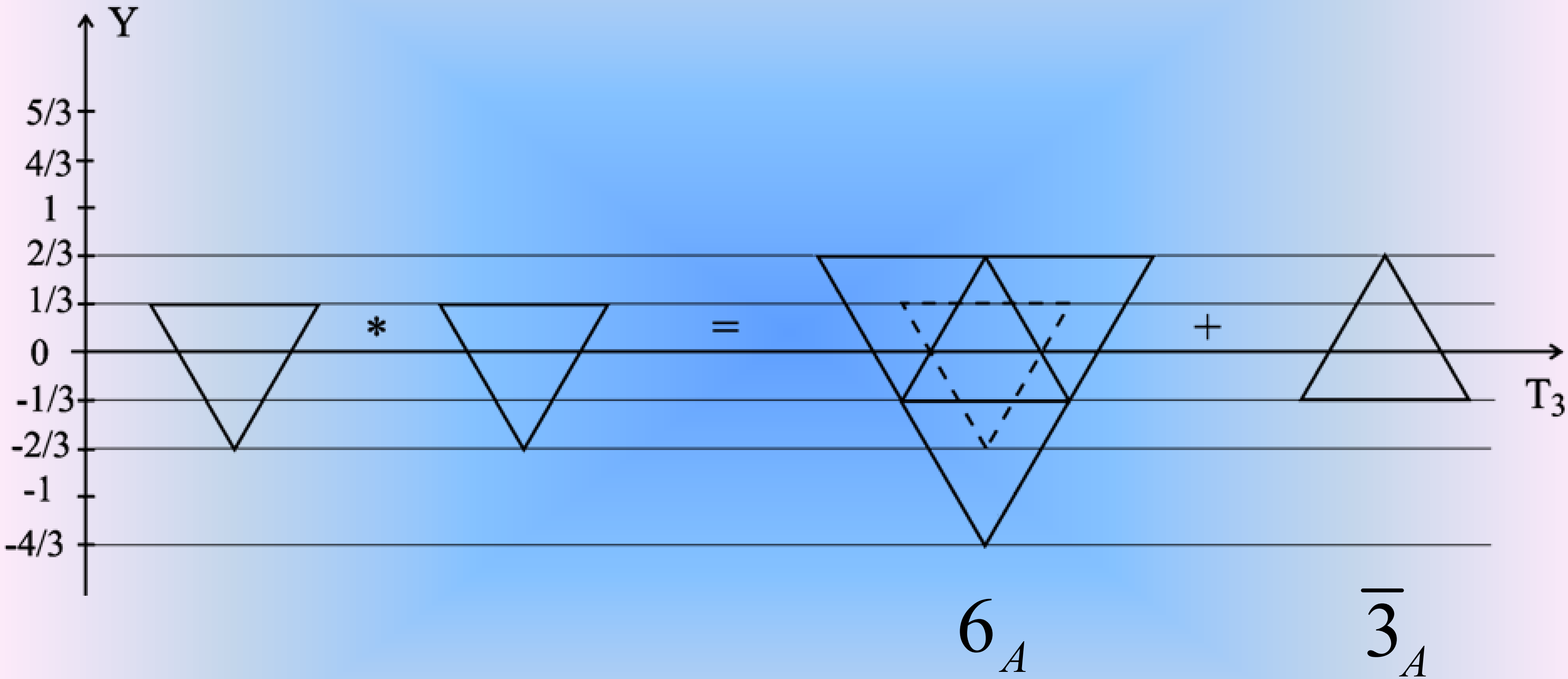
$$3 \times \bar{6} = \overline{15}_s + \bar{3}_s$$

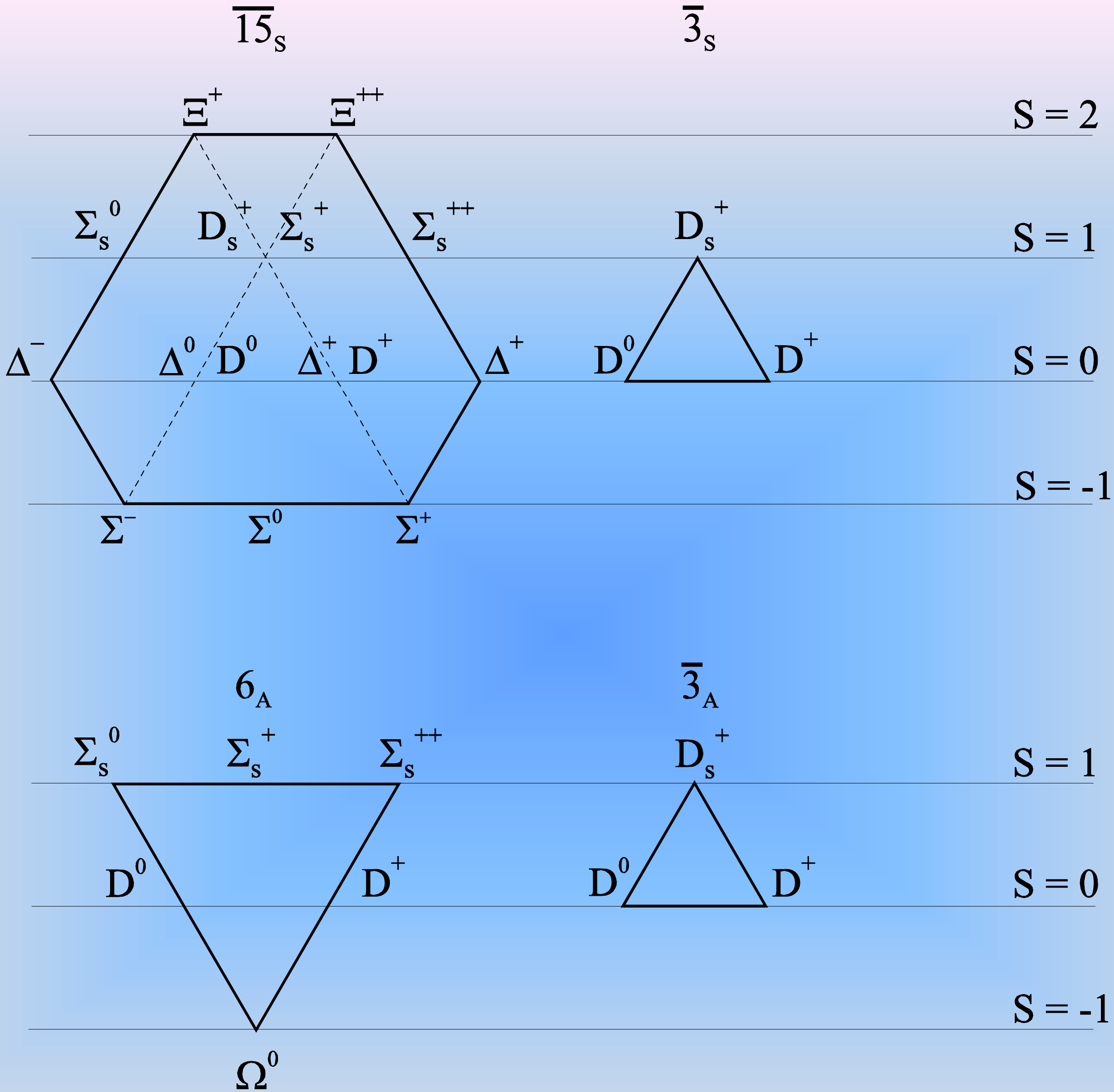



Then we add the third quark triplet.

# WEIGHT DIAGRAMS FOR $q\,\bar{q}\,\bar{q}$ (III)

$3 \times \underline{\underline{3}} = 6_A + \bar{3}_A$






Symmetric and antisymmetric tetraquark multiplets, with label for each tetraquark and with given strangeness S.

The flavor wave functions of scalar $cq\overline{qq}$ tetraquarks distributed in $SU(3)_F$ multiplets $\overline{15}_S$ and $\overline{3}_S$, with mixing between states with the same quantum numbers.

| multiplet | tetraquark | flavor wave function |
|---|---|---|
| $\overline{15}_S$ | $\Xi^{++}$ | $cu\bar{s}\bar{s}$ |
| | $\Xi^{+}$ | $cd\bar{s}\bar{s}$ |
| | $\Sigma_s^{++}$ | $-\frac{1}{\sqrt{2}}cu\left(\bar{d}\bar{s}+\bar{s}\bar{d}\right)$ |
| | $\Sigma_s^{+}$ | $\frac{1}{2}c\left(u\left(\bar{s}\bar{u}+\bar{u}\bar{s}\right)-d\left(\bar{d}\bar{s}+\bar{s}\bar{d}\right)\right)$ |
| | $\Sigma_s^{0}$ | $\frac{1}{\sqrt{2}}cd\left(\bar{s}\bar{u}+\bar{u}\bar{s}\right)$ |
| | $\Delta^{++}$ | $cu\bar{d}\bar{d}$ |
| | $\Delta^{+}$ | $\frac{1}{\sqrt{3}}c\left(-u\left(\bar{u}\bar{d}+\bar{d}\bar{u}\right)+d\bar{d}\bar{d}\right)$ |
| | $\Delta^{0}$ | $\frac{1}{\sqrt{3}}c\left(-d\left(\bar{u}\bar{d}+\bar{d}\bar{u}\right)+u\bar{u}\bar{u}\right)$ |
| | $\Delta^{-}$ | $cd\bar{u}\bar{u}$ |
| | $\Sigma^{+}$ | $cs\bar{d}\bar{d}$ |
| | $\Sigma_s^{0}$ | $-\frac{1}{\sqrt{2}}cs\left(\bar{u}\bar{d}+\bar{d}\bar{u}\right)$ |
| | $\Sigma_s^{-}$ | $cs\bar{u}\bar{u}$ |
| $\overline{15}_S$ and $\bar{3}_S$ mixed states | $D_s^+(\overline{15}_S-\bar{3}_S)$ | $\frac{1}{2\sqrt{2}}c\left(u\left(\bar{s}\bar{u}+\bar{u}\bar{s}\right)+d\left(\bar{d}\bar{s}+\bar{s}\bar{d}\right)-2s\bar{s}\bar{s}\right);\ \frac{1}{2\sqrt{2}}c\left(u\left(\bar{s}\bar{u}+\bar{u}\bar{s}\right)+d\left(\bar{d}\bar{s}+\bar{s}\bar{d}\right)+2s\bar{s}\bar{s}\right)$ |
| | $D^+(\overline{15}_S-\bar{3}_S)$ | $\frac{1}{2\sqrt{6}}c\left(u\left(\bar{u}\bar{d}+\bar{d}\bar{u}\right)-3s\left(\bar{d}\bar{s}+\bar{s}\bar{d}\right)+2d\bar{d}\bar{d}\right);\ \frac{1}{2\sqrt{2}}c\left(u\left(\bar{u}\bar{d}+\bar{d}\bar{u}\right)+s\left(\bar{d}\bar{s}+\bar{s}\bar{d}\right)+2d\bar{d}\bar{d}\right)$ |
| | $D^0(\overline{15}_S-\bar{3}_S)$ | $\frac{1}{2\sqrt{6}}c\left(-d\left(\bar{u}\bar{d}+\bar{d}\bar{u}\right)+3s\left(\bar{s}\bar{u}+\bar{u}\bar{s}\right)-2u\bar{u}\bar{u}\right);\ \frac{1}{2\sqrt{2}}c\left(d\left(\bar{u}\bar{d}+\bar{d}\bar{u}\right)+s\left(\bar{s}\bar{u}+\bar{u}\bar{s}\right)+2u\bar{u}\bar{u}\right)$ |

The flavor wave functions of scalar $cq\overline{qq}$ tetraquarks distributed in $SU(3)_F$ multiplets $6_A$ and $\overline{3}_A$, with mixing between states with the same quantum numbers.

| | | |
|---|---|---|
| $6_A$ | $\Sigma_s^{++}$<br>$\Sigma_s^{+}$<br>$\Sigma_s^{0}$<br>$\Omega^0$ | $\frac{1}{\sqrt{2}}cu\left(\bar{d}\bar{s}-\bar{s}\bar{d}\right)$<br>$\frac{1}{2}c\left(u\left(\bar{s}\bar{u}-\bar{u}\bar{s}\right)+d\left(\bar{d}\bar{s}-\bar{s}\bar{d}\right)\right)$<br>$\frac{1}{\sqrt{2}}cd\left(\bar{s}\bar{u}-\bar{u}\bar{s}\right)$<br>$\frac{1}{\sqrt{2}}cs\left(\bar{u}\bar{d}-\bar{d}\bar{u}\right)$ |
| $\bar{3}_A$ | $D_s^{+}$ | $\frac{1}{2}c\left(d\left(\bar{d}\bar{s}-\bar{s}\bar{d}\right)-u\left(\bar{s}\bar{u}-\bar{u}\bar{s}\right)\right)$ |
| $6_A$ and $\bar{3}_A$ mixed states | $D^{+}(6_A-\bar{3}_A)$<br>$D^{0}(6_A-\bar{3}_A)$ | $\frac{1}{2}c\left(u\left(\bar{u}\bar{d}-\bar{d}\bar{u}\right)+s\left(\bar{d}\bar{s}-\bar{s}\bar{d}\right)\right)$; $\frac{1}{2}c\left(u\left(\bar{u}\bar{d}-\bar{d}\bar{u}\right)-s\left(\bar{d}\bar{s}-\bar{s}\bar{d}\right)\right)$<br>$\frac{1}{2}c\left(d\left(\bar{u}\bar{d}-\bar{d}\bar{u}\right)+s\left(\bar{s}\bar{u}-\bar{u}\bar{s}\right)\right)$; $\frac{1}{2}c\left(s\left(\bar{s}\bar{u}-\bar{u}\bar{s}\right)-d\left(\bar{u}\bar{d}-\bar{d}\bar{u}\right)\right)$ |

In our model (total spin = 0), the corresponding symmetric $\chi_S$ and antisymmetric $\chi_A$ spin functions have the form:

$$|\chi_S\rangle = \frac{-1}{2\sqrt{3}}|\uparrow\downarrow\uparrow\downarrow + \downarrow\uparrow\uparrow\downarrow + \uparrow\downarrow\downarrow\uparrow + \downarrow\uparrow\downarrow\uparrow - 2\uparrow\uparrow\downarrow\downarrow - 2\downarrow\downarrow\uparrow\uparrow\rangle$$

$$|\chi_A\rangle = \frac{1}{2}|\uparrow\downarrow\uparrow\downarrow - \downarrow\uparrow\uparrow\downarrow - \uparrow\downarrow\downarrow\uparrow + \downarrow\uparrow\downarrow\uparrow\rangle$$

As it is known for fermions (spin 1/2) the total wave function has to be antisymmetric and the color state function is antisymmetric, then we conclude:

the particles from multiplets $\overline{15}_S$ and $\overline{3}_S$ have the symmetric spin and flavor wave functions ($\chi_S$ $\nu_S$), while the particles from multiplets $6_A$ and $\overline{3}_A$ have the antisymmetric spin and flavor wave functions ($\chi_A$ $\nu_A$).

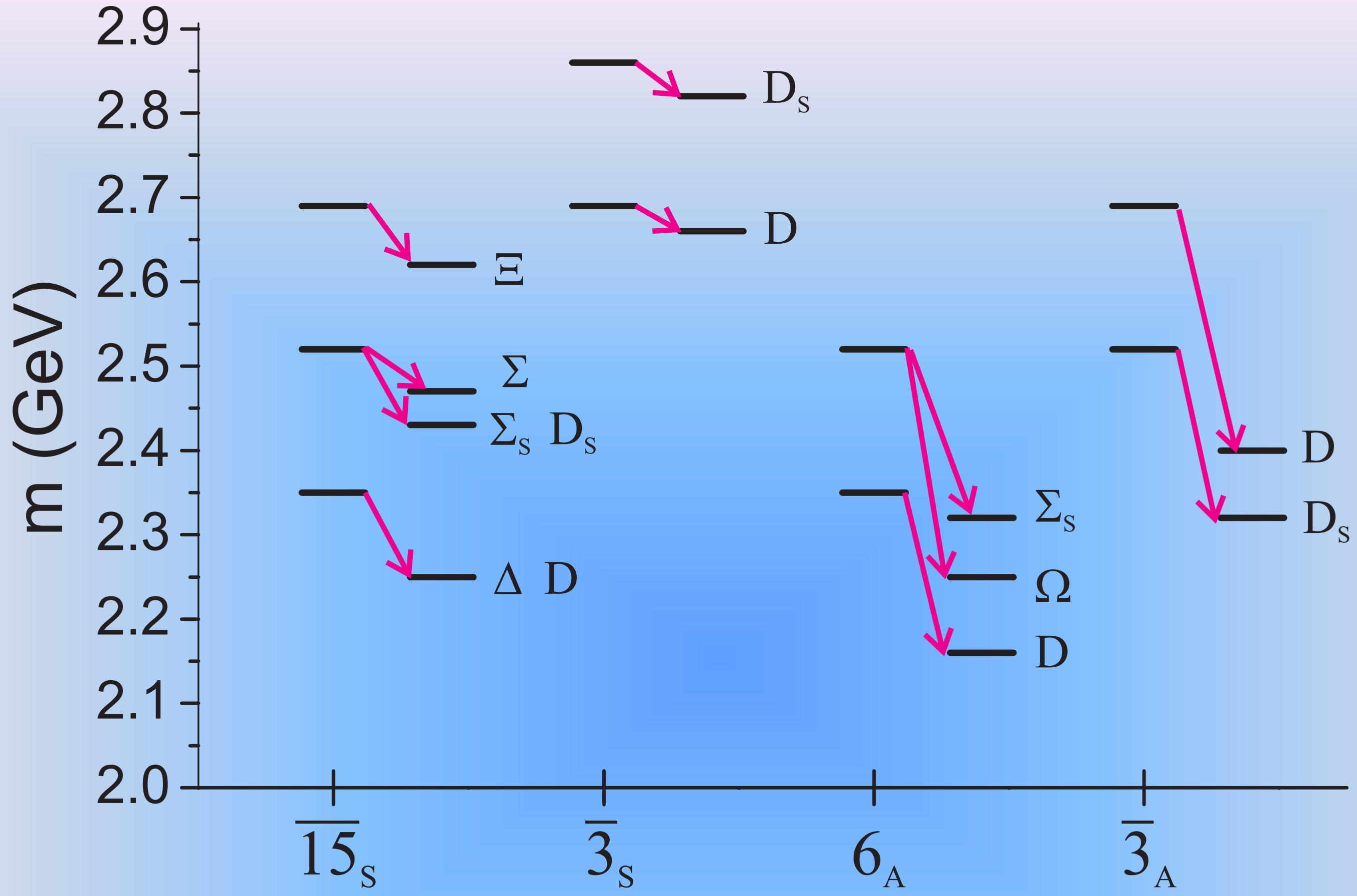
Masses of scalar $cq\overline{qq}$ tetraquarks distributed in $SU(3)_F$ multiplets, with mixing between states with the same quantum numbers. $m_{\nu,0}$ are tetraquark masses without influence of FB HFI and $m_{\nu,FB}$ are FB HFI contributions to tetraquark masses.

| tetraquark | $m_{\nu,0}$ | $m_{\nu,FB}$ ($m_u = m_d$) | $m_\nu$ (GeV) |
|---|---|---|---|
| multiplet $\overline{15}_S$ | | | |
| $\Xi$ | $m_u + 2m_s + m_c$ | $\frac{8}{3}C\left(\frac{2}{m_S m_C}+\frac{2}{m_U m_S}-\frac{1}{m_U m_C}-\frac{1}{m_S{}^2}\right)$ | 2.62 |
| $\Sigma_s$ | $2m_u + m_s + m_c$ | $\frac{8}{3}C\left(\frac{1}{m_U{}^2}+\frac{1}{m_S m_C}\right)$ | 2.43 |
| $\Delta$ | $3m_u + m_c$ | $\frac{8}{3}C\left(\frac{1}{m_U{}^2}+\frac{1}{m_U m_C}\right)$ | 2.25 |
| $\Sigma$ | $2m_u + m_s + m_c$ | $\frac{8}{3}C\left(\frac{2}{m_U m_C}+\frac{2}{m_U m_S}-\frac{1}{m_S m_C}-\frac{1}{m_U{}^2}\right)$ | 2.47 |

$m_u = 310$ MeV, $m_s = 480$ MeV, $m_c = 1419$ MeV constituent quark masses

$C^{meson} = 1.5 \cdot 10^7$ MeV$^3$; $C^{tetra} = C^{meson} / (-16/3)$ constant

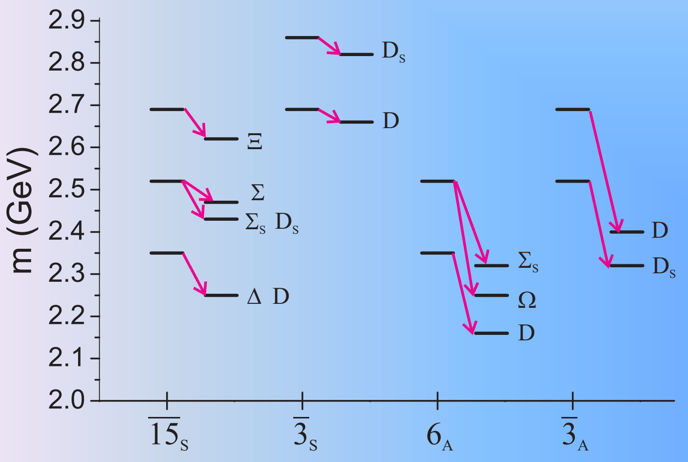
| | | | |
|---|---|---|---|
| $\overline{15}_S$ and $\overline{3}_S$ mixed states | | | |
| $D_s\ (\overline{15}_S - \overline{3}_S)$ | $2m_u + m_s + m_c$<br>$3m_s + m_c$ | $\frac{8}{3}C\left(\frac{1}{m_U^2}+\frac{1}{m_S m_C}\right);\frac{8}{3}C\left(\frac{1}{m_S^2}+\frac{1}{m_S m_C}\right)$ | 2.43<br>2.82 |
| $D\ (\overline{15}_S - \overline{3}_S)$ | $3m_u + m_c$<br>$m_u + 2m_s + m_c$ | $\frac{8}{3}C\left(\frac{1}{m_U^2}+\frac{1}{m_U m_C}\right);\frac{4}{3}C\left(\frac{3}{m_S^2}+\frac{2}{m_U m_C}-\frac{1}{m_U^2}\right)$ | 2.25<br>2.66 |
| multiplet $6_A$ | | | |
| $\Sigma_s$ | $2m_u + m_s + m_c$ | $8C\left(\frac{1}{m_U m_S}+\frac{1}{m_U m_C}\right)$ | 2.32 |
| $\Omega$ | $2m_u + m_s + m_c$ | $8C\left(\frac{1}{m_U^2}+\frac{1}{m_S m_C}\right)$ | 2.25 |
| multiplet $\overline{3}_A$ | | | |
| $D_s$ | $2m_u + m_s + m_c$ | $8C\left(\frac{1}{m_U m_S}+\frac{1}{m_U m_C}\right)$ | 2.32 |
| $6_A$ and $\overline{3}_A$ mixed states | | | |
| $D\ (6_A - \overline{3}_A)$ | $3m_u + m_c$<br>$m_u + 2m_s + m_c$ | $8C\left(\frac{1}{m_U m_S}+\frac{1}{m_S m_C}\right);8C\left(\frac{1}{m_U^2}+\frac{1}{m_U m_C}\right)$ | 2.16<br>2.40 |

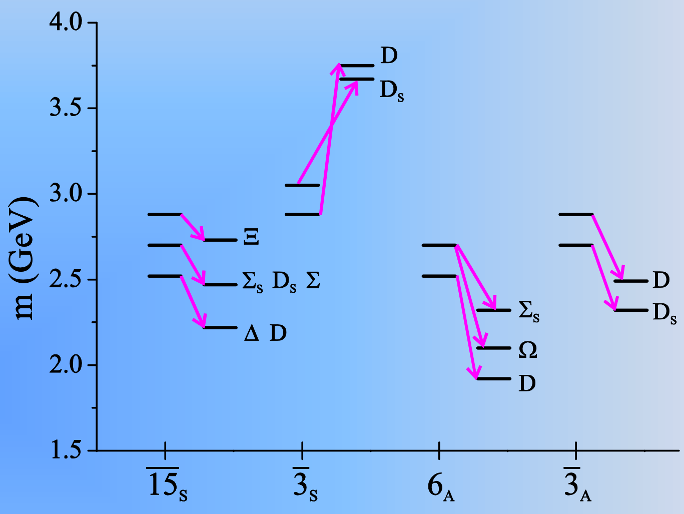



Tetraquark mass spectrum without (left column) and with (right column) FB HFI, both with $SU(3)_F$ symmetry breaking.

FB HFI reduces the obtained masses and causes splitting between $\Sigma$ and $\Sigma_s$ in $\overline{15}_S$ and between $\Sigma_s$ and $\Omega$ in $6_A$.

FB HFI (color-spin) GR HFI (flavor-spin)




From this comparison one can see that the forms of tetraquark spectra with FB and GR interaction are similar, only they are shifted for some value. This result was not expected because FB is color-spin and GR is flavor-spin interaction.

# $D_s^+(2317)$, $D_s^+(2632)$ and $D^0(2308)$ states

Obtained pattern indicates that $D_s^+(2317)$ (BABAR Collab. 2003, PRL **90**, 242001) is a four-quark state in the $SU(3)_F$ $\overline{3}_A$ representation with the quark content:

$$|cq\overline{qs}\rangle = \frac{1}{2}c\left(d\left(\overline{ds}-\overline{sd}\right)-u\left(\overline{su}-\overline{us}\right)\right)$$

$D_s^+(2632)$ (SELEX Collab. 2004, PRL **93**, 242001) is identified with the state from mixing of the $\overline{15}_S$ and $\overline{3}_S$ representations:

$$|cq\overline{qs}\rangle = \frac{1}{2\sqrt{2}}c\left(u\left(\overline{su}+\overline{us}\right)+d\left(\overline{ds}+\overline{sd}\right)-2s\overline{ss}\right);$$

$$|cq\overline{qs}\rangle = \frac{1}{2\sqrt{2}}c\left(u\left(\overline{su}+\overline{us}\right)+d\left(\overline{ds}+\overline{sd}\right)-2s\overline{ss}\right)$$

$D^0$(2308) (BELLE Collab. 2004, PRD **69**, 112002) is identified with the state from mixing of the $6_A$ and $\overline{3}_A$ representations:

$$\left|cq\overline{qs}\right\rangle = \frac{1}{2}c\left(d\left(\overline{ud} - \overline{du}\right) + s\left(\overline{su} - \overline{us}\right)\right);$$

$$\left|cq\overline{qs}\right\rangle = \frac{1}{2}c\left(s\left(\overline{su} - \overline{us}\right) - d\left(\overline{ud} - \overline{du}\right)\right)$$

# Conclusions

- ✓ We confirmed tetraquark nature for scalar charmed mesons, i.e. we showed existence of tetraquark component in their wave functions.
- ✓ We have made a systematic analysis of the charm tetraquark states. Weight diagrams, irreducible representations and flavor wave functions are shown and analyzed.
- ✓ symmetric and antisymmetric spin wave functions are given
- ✓ Mass spectra with mixing of particles with the same quantum numbers are shown.
- ✓ Mixing splits the two states
- ✓ More experimental searches for detection of other members are needed in the future.

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