Minkowskian metrics as a consequence of generalized supersymmetry

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Poincare algebra

$[J_i, J_j] = i\,\varepsilon_{ijk}\,J_k,$ $[J_i, N_j] = i\,\varepsilon_{ijk}\,N_k,$

$[N_i, N_j] = -i\,\varepsilon_{ijk}\,J_k,$ $[J_i, P_j] = i\,\varepsilon_{ijk}\,P_k,$

$[N_i, P_j] = i\,\delta_{ij}\,P_0,$ $[N_i, P_0] = i\,P_i,$

$[P_i, P_j] = 0,$ $[P_i, P_0] = 0.$

$[M_{\mu\nu}, M_{\lambda\rho}] = i\,(\eta_{\nu\lambda} M_{\mu\rho} + \eta_{\mu\rho} M_{\nu\lambda} - \eta_{\mu\lambda} M_{\nu\rho} - \eta_{\nu\rho} M_{\mu\lambda}),$

$[M_{\mu\nu}, P_\lambda] = i\,(\eta_{\nu\lambda} P_\mu + \eta_{\mu\lambda} P_\nu),$ $[P_\mu, P_\nu] = 0$

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Poincare superalgebra

$[M_{\mu\nu}, M_{\lambda\rho}] = i\,(\eta_{\nu\lambda} M_{\mu\rho} + \eta_{\mu\rho} M_{\nu\lambda} - \eta_{\mu\lambda} M_{\nu\rho} - \eta_{\nu\rho} M_{\mu\lambda}),$

$[M_{\mu\nu}, P_{\lambda}] = i\,(\eta_{\nu\lambda} P_{\mu} + \eta_{\mu\lambda} P_{\nu}),$ $\qquad [P_{\mu}, P_{\nu}] = 0$

$[M_{\mu\nu}, Q_{\alpha}] = -1/4\,([\gamma_{\mu}, \gamma_{\nu}])_{\alpha}{}^{\beta} Q_{\beta},$

$\{Q_{\alpha}, \bar{Q}_{\beta}\} = -2i\,(\gamma^{\mu})_{\alpha\beta} P_{\mu},$ $\qquad [P_{\mu}, Q_{\alpha}] = 0$

$[M_{\mu\nu}, S_{\alpha}] = -1/4\,([\gamma_{\mu}, \gamma_{\nu}])_{\alpha}{}^{\beta} S_{\beta},$

$\{S_{\alpha}, S_{\beta}\} = -2i\,(\gamma^{\mu})_{\alpha\beta} K_{\mu},$

$[K_{\mu}, S_{\alpha}] = 0,$

etc…

Conformal superalgebra

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

But…

- There is no experimental data on (spacetime) supersymmetry, so how do we so surely know that it should be of the standard Poencare (conformal) type?

  - We don’t! (HLS presumptions are over constraining)

- Yet, why is 99.9% of supersymmetry work and of empirical predictions based on (N-dimensional) Poencare (conformal) susy?

  - Indeed, why???

Is this really necessary?

$\{Q_\alpha, Q_\beta\} = 0$

$\{Q_\alpha, S_{\dot\beta}\} = 0$

$\{S_\alpha, S_\beta\} = 0$

$\{Q_{\dot\alpha}, Q_{\dot\beta}\} = 0$ (=central)

$\{Q_{\dot\alpha}, S_\beta\} = 0$ (=central)

$\{S_{\dot\alpha}, S_{\dot\beta}\} = 0$ (=central)

generalized supersymmetry

Aim of this talk is to show:

- Generalized supersymmetry is mathematically simpler and yet richer way to express supersymmetry idea:
  - Parabose N=4 algebra can be seen as extension of conformal supersymmetry
  - A simple symmetry breaking [essentially SU(2) to U(1)] can reduce this symmetry down to Poincare symmetry
- Minkowsky metric is not postulated in this approach, but follows from the symmetry breaking
- Even without symmetry breaking time axis stands out from the rest of the axes

Parabose algebra

- Start with 4 pairs of operators a and a⁺ satisfying:

$$[\{\hat{a}_\alpha, \hat{a}_\beta\}, \hat{a}_\gamma] = 0, \qquad [\{\hat{a}_\alpha, \hat{a}^\dagger_\beta\}, \hat{a}_\gamma] = -2\delta^\gamma_\beta \hat{a}_\alpha$$

- Switch to hermitian combinations:

$$S^\alpha \equiv (\hat{a}_\alpha + \hat{a}^\dagger_\alpha), \qquad Q_\alpha \equiv -i(\hat{a}_\alpha - \hat{a}^\dagger_\alpha).$$

consequently satisfying:

$$[\{Q_\alpha, Q_\beta\}, Q_\gamma] = 0,$$
$$[\{Q_\alpha, Q_\beta\}, S^\gamma] = -4i\delta^\gamma_\beta Q_\alpha - 4i\delta^\gamma_\alpha Q_\beta,$$
$$[\{S^\alpha, Q_\beta\}, S^\gamma] = 4i\delta^\gamma_\beta S^\alpha,$$

$$[\{S^\alpha, S^\beta\}, S^\gamma] = 0,$$
$$[\{S^\alpha, S^\beta\}, Q_\gamma] = 4i\delta^\beta_\gamma S^\alpha + 4i\delta^\alpha_\gamma S^\beta,$$
$$[\{Q_\alpha, S^\beta\}, Q_\gamma] = 4i\delta^\beta_\gamma Q_\alpha.$$

Basis of 4 by 4 real matrices

6 antisymmetric matrices:

$$[\sigma_i, \sigma_j] = 2\varepsilon_{ijk}\sigma_k$$
$$[\tau_{\underline{i}}, \tau_{\underline{j}}] = 2\varepsilon_{\underline{i}\underline{j}\underline{k}}\tau_{\underline{k}}$$
$$[\sigma_i, \tau_{\underline{j}}] = 0$$

$$\sigma_1 = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix},$$

$$\tau_1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}.$$

10 symmetric matrices:

$$\alpha_{\underline{i}j} \equiv \tau_{\underline{i}}\sigma_j$$
$$\alpha_0 = 1$$

$$\alpha_{11} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \alpha_{12} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad \alpha_{13} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix},$$

$$\alpha_{21} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \quad \alpha_{22} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \alpha_{23} = -\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix},$$

$$\alpha_{31} = -\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad \alpha_{32} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \quad \alpha_{33} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

New basis for expressing parabose anticommutators

$$\hat{J}_i \equiv \frac{1}{8}(\sigma_i)^{\alpha}{}_{\beta}\{Q_\alpha, S^\beta\}, \qquad Y_{\underline{i}} \equiv \frac{1}{8}(\tau_{\underline{i}})^{\alpha}{}_{\beta}\{Q_\alpha, S^\beta\},$$

$$\hat{N}_{\underline{i}j} \equiv \frac{1}{8}(\alpha_{\underline{i}j})^{\alpha}{}_{\beta}\{Q_\alpha, S^\beta\}, \qquad \hat{D} \equiv (\alpha_0)^{\alpha}{}_{\beta}\{Q_\alpha, S^\beta\},$$

$$\hat{P}_{\underline{i}j} \equiv \frac{1}{8}(\alpha_{\underline{i}j})^{\alpha\beta}\{Q_\alpha, Q_\beta\}, \qquad \hat{P}_0 \equiv \frac{1}{8}(\alpha_0)^{\alpha\beta}\{Q_\alpha, Q_\beta\},$$

$$\hat{K}_{\underline{i}j} \equiv -\frac{1}{8}(\alpha_{\underline{i}j})_{\alpha\beta}\{S^\alpha, S^\beta\}, \qquad \hat{K}_0 \equiv \frac{1}{8}(\alpha_0)_{\alpha\beta}\{S^\alpha, S^\beta\},$$

Starting parabose relations obtain a new form:

$$[\{\hat{a}_\alpha, \hat{a}_\beta\}, \hat{a}_\gamma] = 0, \qquad [\{\hat{a}_\alpha, \hat{a}^\dagger_\beta\}, \hat{a}_\gamma] = -2\delta^\gamma_\beta \hat{a}_\alpha$$

$$\Downarrow$$

$$\begin{array}{lll}
[\hat{J}_i, Q_\alpha] = -i(\frac{\sigma_i}{2})_\alpha{}^\beta Q_\beta, & [Y_{\underline{i}}, Q_\alpha] = -i(\frac{\tau_{\underline{i}}}{2})_\alpha{}^\beta Q_\beta, & [\hat{N}_{\underline{i}j}, Q_\alpha] = i(\frac{\alpha_{\underline{i}j}}{2})_\alpha{}^\beta Q_\beta, \\
[\hat{J}_i, S^\alpha] = -i(\frac{\sigma_i}{2})^\alpha{}_\beta S^\beta, & [Y_{\underline{i}}, S^\alpha] = -i(\frac{\tau_{\underline{i}}}{2})^\alpha{}_\beta S^\beta, & [\hat{N}_{\underline{i}j}, S^\alpha] = -i(\frac{\alpha_{\underline{i}j}}{2})^\alpha{}_\beta S^\beta, \\
[\hat{K}_0, Q_\alpha] = i(\alpha_0)_{\alpha\beta} S^\beta, & [\hat{K}_{\underline{i}j}, Q_\alpha] = -i(\alpha_{\underline{i}j})_{\alpha\beta} S^\beta, & [\hat{K}_0, S^\alpha] = [\hat{K}_{\underline{i}j}, S^\alpha] = 0, \\
[\hat{P}_0, S^\alpha] = -i(\alpha_0)^{\alpha\beta} Q_\beta, & [\hat{P}_{\underline{i}j}, S^\alpha] = -i(\alpha_{\underline{i}j})^{\alpha\beta} Q_\beta, & [\hat{P}_0, Q_\alpha] = [\hat{P}_{\underline{i}j}, Q_\alpha] = 0, \\
[\hat{D}, Q_\alpha] = i(\frac{1}{2}) Q_\alpha, & [\hat{D}, S^\alpha] = -i(\frac{1}{2}) S^\alpha. &
\end{array}$$

Algebra of anticommutators

$$[\hat J_i, \hat J_j] = i\,\varepsilon_{ijk}\hat J_k, \qquad [\hat Y_{\underline{i}}, \hat Y_{\underline{j}}] = i\,\varepsilon_{\underline{ijk}}\hat Y_{\underline{k}},$$
$$[\hat J_i, \hat Y_{\underline{j}}] = 0.$$

Isomorphic to *sp*(8)

$$[\hat J_i, \hat N_{\underline{j}k}] = i\varepsilon_{ikl}\hat N_{\underline{j}l}, \qquad [\hat Y_{\underline{i}}, \hat N_{\underline{j}k}] = i\varepsilon_{ijl}\hat N_{\underline{l}k},$$
$$[\hat N_{\underline{i}j}, \hat N_{\underline{k}l}] = -i\left(\delta_{jl}\varepsilon_{\underline{ikm}}\hat Y_{\underline{m}} + \delta_{\underline{ik}}\varepsilon_{jlm}\hat J_m\right),$$
$$[\hat J_i, \hat D] = [\hat Y_{\underline{i}}, \hat D] = [\hat N_{\underline{i}j}, \hat D] = 0.$$

$$[\hat J_i, \hat P_{\underline{j}k}] = i\varepsilon_{ikl}\hat P_{\underline{j}l}, \qquad [\hat Y_{\underline{i}}, \hat P_{\underline{j}k}] = i\varepsilon_{\underline{ijl}}\hat P_{\underline{l}k},$$
$$[\hat N_{\underline{i}j}, \hat P_{\underline{k}l}] = i\delta_{\underline{ik}}\delta_{jl}\hat P_0 + i\varepsilon_{\underline{ikm}}\varepsilon_{jln}\hat P_{\underline{m}n},$$
$$[\hat N_{\underline{i}j}, \hat P_0] = i\hat P_{\underline{i}j}, \qquad [\hat D, \hat P_{\underline{i}j}] = i\hat P_{\underline{i}j},$$
$$[\hat D, \hat P_0] = i\hat P_0, \qquad [\hat J_i, \hat P_0] = [\hat Y_{\underline{i}}, \hat P_0] = 0.$$

$$[\hat J_i, \hat K_{\underline{j}k}] = i\varepsilon_{ikl}\hat K_{\underline{j}l}, \qquad [\hat Y_{\underline{i}}, \hat K_{\underline{j}k}] = i\varepsilon_{\underline{ijl}}\hat K_{\underline{l}k}, \qquad \ldots$$
$$[\hat P_{\underline{i}j}, \hat K_{\underline{k}l}] = 2i\left(-\delta_{\underline{ik}}\delta_{jl}\hat D - \varepsilon_{\underline{ikm}}\varepsilon_{jln}\hat N_{\underline{m}n} + \delta_{\underline{ik}}\varepsilon_{jlm}\hat J_m + \delta_{jl}\varepsilon_{\underline{ikm}}\hat Y_{\underline{m}}\right),$$
$$[\hat P_{\underline{i}j}, \hat K_0] = -2i\hat N_{\underline{i}j}, \qquad [\hat P_0, \hat K_{\underline{i}j}] = -2i\hat N_{\underline{i}j},$$
$$[\hat P_0, \hat K_0] = -2i\hat D,$$

Symmetry breaking

D	J_1	J_2	J_3
Y_1	N_{11}	N_{12}	N_{13}
Y_2	N_{21}	N_{22}	N_{23}
Y_3	N_{31}	N_{32}	N_{33}

{Q,S} operators

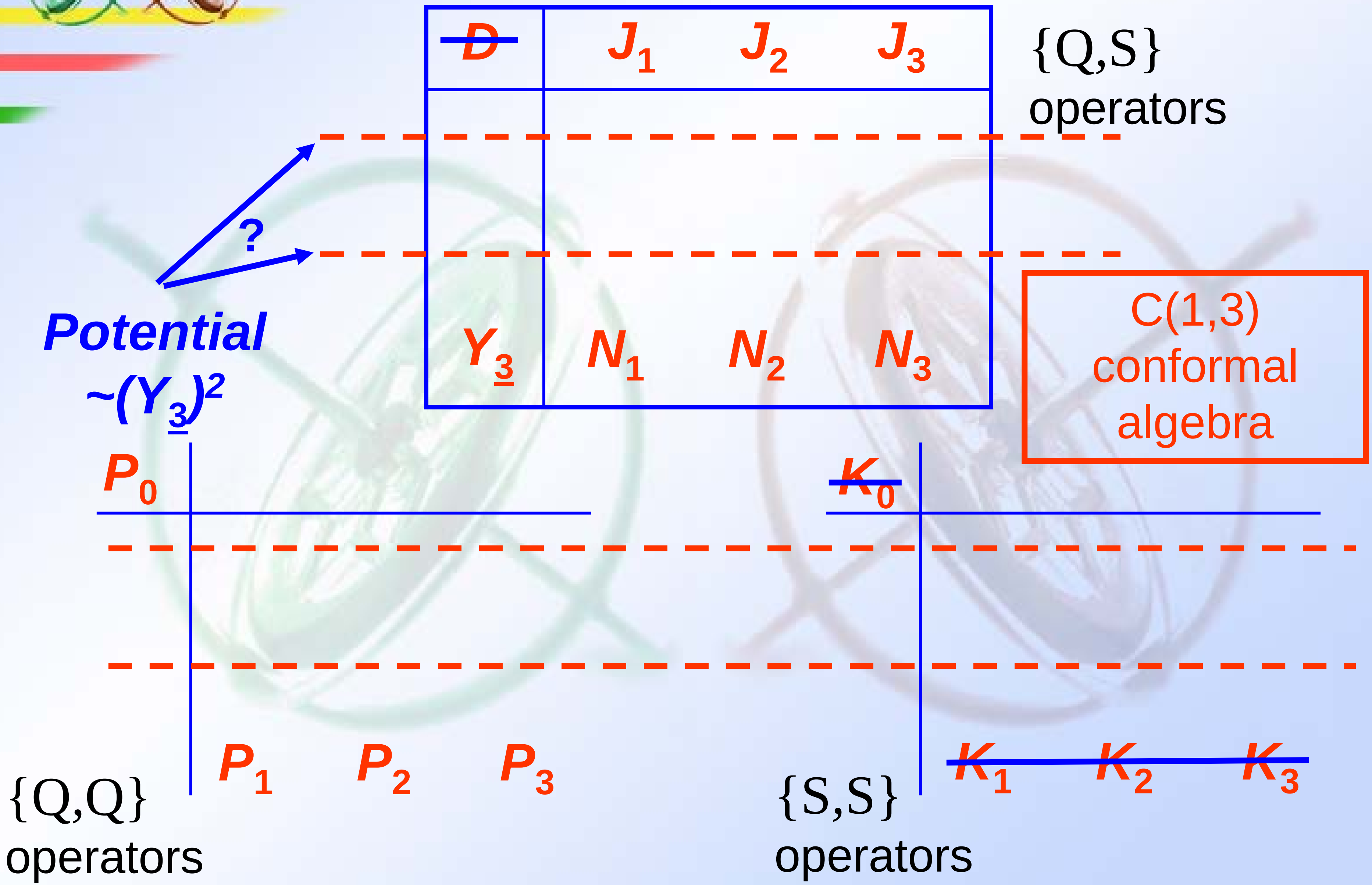
P_0			
	P_{11}	P_{12}	P_{13}
	P_{21}	P_{22}	P_{23}
	P_{31}	P_{32}	P_{33}

{Q,Q} operators

K_0			
	K_{11}	K_{12}	K_{13}
	K_{21}	K_{22}	K_{23}
	K_{31}	K_{32}	K_{33}

{S,S} operators

Symmetry breaking



Full bosonic symmetry

D	J_1	J_2	J_3
Y_1	N_{11}	N_{12}	N_{13}
Y_2	N_{21}	N_{22}	N_{23}
Y_3	N_{31}	N_{32}	N_{33}

{Q,S} operators

$$\hat{P}_0 = \frac{1}{2}(\hat{Q}_1^2 + \hat{Q}_2^2 + \hat{Q}_3^2 + \hat{Q}_4^2)$$

P_0			
	P_{11}	P_{12}	P_{13}
	P_{21}	P_{22}	P_{23}
	P_{31}	P_{32}	P_{33}

{Q,Q} operators

$$\hat{P}_{31} = -\hat{Q}_1\hat{Q}_2 - \hat{Q}_3\hat{Q}_4$$

$$\hat{P}_{32} = \hat{Q}_1\hat{Q}_4 - \hat{Q}_2\hat{Q}_3$$

$$\hat{P}_{33} = \frac{1}{2}(-\hat{Q}_1^2 + \hat{Q}_2^2 - \hat{Q}_3^2 + \hat{Q}_4^2)$$

	K_{31}	K_{32}	K_{33}

{S,S} operators

“Extended” conformal superalgebra

$$[\{\hat{a}_\alpha, \hat{a}_\beta\}, \hat{a}_\gamma] = 0, \quad [\{\hat{a}_\alpha, \hat{a}^\dagger_\beta\}, \hat{a}_\gamma] = -2\delta^\gamma_\beta \hat{a}_\alpha$$

$$\{Q_\alpha, Q_\beta\} = (\alpha_0)_{\alpha\beta}\, P_0 + (\alpha_{\underline{i}j})_{\alpha\beta}\, P \ldots$$

$$\{S^\alpha, S^\beta\} = (\alpha_0)^{\alpha\beta}\, K_0 - (\alpha_{\underline{i}j})^{\alpha\beta}\, K \ldots$$

$$\{S^\alpha, Q_\beta\} = (\alpha_0)^\alpha{}_\beta\, D + (\alpha_{\underline{i}j})^\alpha{}_\beta\, N_{\underline{i}} \ldots$$

of

sis

$$[\hat{J}_i, \hat{J}_j] = i\,\varepsilon_{ijk}\hat{J}_k, \quad [\hat{Y}_{\underline{i}}, \hat{Y}_{\underline{j}}] = i\,\varepsilon_{\underline{ijk}}\hat{Y}_{\underline{k}},$$
$$[\hat{J}_i, \hat{Y}_{\underline{j}}] = 0.$$

$$[\hat{J}_i, \hat{N}_{\underline{j}k}] = i\varepsilon_{ikl}\hat{N}_{\underline{j}l}, \qquad [\hat{Y}_{\underline{i}}, \hat{N}_{\underline{j}k}] = i\varepsilon_{\underline{ijl}}\hat{N}_{\underline{l}k},$$
$$[\hat{N}_{\underline{i}j}, \hat{N}_{\underline{k}l}] = -i\left(\delta_{jl}\varepsilon_{\underline{ikm}}\hat{Y}_{\underline{m}} + \delta_{\underline{ik}}\varepsilon_{jlm}\hat{J}_m\right),$$
$$[\hat{J}_i, \hat{D}] = [\hat{Y}_{\underline{i}}, \hat{D}] = [\hat{N}_{\underline{i}j}, \hat{D}] = 0.$$

$$[\hat{J}_i, \hat{P}_{\underline{j}k}] = i\varepsilon_{ikl}\hat{P}_{\underline{j}l}, \qquad [\hat{Y}_{\underline{i}}, \hat{P}_{\underline{j}k}] = i\varepsilon_{\underline{ijl}}\hat{P}_{\underline{l}k},$$
$$[\hat{N}_{\underline{i}j}, \hat{P}_{\underline{k}l}] = i\delta_{\underline{ik}}\delta_{jl}\hat{P}_0 + i\varepsilon_{\underline{ikm}}\varepsilon_{jln}\hat{P}_{\underline{m}n},$$
$$[\hat{N}_{\underline{i}j}, \hat{P}_0] = i\hat{P}_{\underline{i}j}, \qquad [\hat{D}, \hat{P}_{\underline{i}j}] = i\hat{P}_{\underline{i}j},$$
$$[\hat{D}, \hat{P}_0] = i\hat{P}_0, \qquad [\hat{J}_i, \hat{P}_0] = [\hat{Y}_{\underline{i}}, \hat{P}_0] = 0.$$

$$[\hat{J}_i, \hat{K}_{\underline{j}k}] = i\varepsilon_{ikl}\hat{K}_{\underline{j}l}, \qquad [\hat{Y}_{\underline{i}}, \hat{K}_{\underline{j}k}] = i\varepsilon_{\underline{ijl}}\hat{K}_{\underline{l}k}, \qquad \ldots$$
$$[\hat{P}_{\underline{i}j}, \hat{K}_{\underline{k}l}] = 2i\left(-\delta_{\underline{ik}}\delta_{jl}\hat{D} - \varepsilon_{\underline{ikm}}\varepsilon_{jln}\hat{N}_{\underline{m}n} + \delta_{\underline{ik}}\varepsilon_{jlm}\hat{J}_m + \delta_{jl}\varepsilon_{\underline{ikm}}\hat{Y}_{\underline{m}}\right),$$
$$[\hat{P}_{\underline{i}j}, \hat{K}_0] = -2i\hat{N}_{\underline{i}j}, \qquad [\hat{P}_0, \hat{K}_{\underline{i}j}] = -2i\hat{N}_{\underline{i}j},$$
$$[\hat{P}_0, \hat{K}_0] = -2i\hat{D},$$

$$[\hat{J}_i, Q_\alpha] = -i(\tfrac{\sigma_i}{2})_\alpha{}^\beta\, Q_\beta, \quad [Y_{\underline{i}}, Q_\alpha] = \ldots Q_\beta,$$
$$[\hat{J}_i, S^\alpha] = -i(\tfrac{\sigma_i}{2})^\alpha{}_\beta\, S^\beta, \quad [Y_{\underline{i}}, S^\alpha] = \ldots S^\beta,$$
$$[\hat{K}_0, Q_\alpha] = i(\alpha_0)_{\alpha\beta}\, S^\beta, \quad [\hat{K}_{\underline{i}j}, Q_\alpha] \ldots = 0,$$
$$[\hat{P}_0, S^\alpha] = -i(\alpha_0)^{\alpha\beta}\, Q_\beta, \quad [\hat{P}_{\underline{i}j}, S^\alpha] \ldots = 0$$
$$[\hat{D}, Q_\alpha] = i(\tfrac{1}{2})Q_\alpha, \qquad [\hat{D}, S^\alpha] = -i(\tfrac{1}{2})S^\alpha.$$

+ bosonic part of algebra

Conclusion

- By giving up {Q,Q} = 0 (=central) we arrived to a formally simpler but mathematically richer parabose algebra, that is relatively easily broken down to Poincare symmetry
- No need to postulate any metrics (signature)
- This interesting picture that reveals a “space-time” with two rotational groups and one unique “time axis” is obtained by sacrificing manifest Lorentz covariance

Thank you!