# Prostorno-vremensko dejstvo za strunu u zavisnosti od torzije i nemetričnosti

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- Definicija modela i kanonska analiza
  - B. Sazdović Torsion and nonmetricity in the stringy geometry
    (*hep-th/0304086*)
  - Bosonic string theory in background fields by canonical methods
    *IJMP* **A 20** (2005) 5501;

- Geometrijske osobine prostor vremena (koneksija, torzija i nemetričnost)
  - D. S. Popović and B. Sazdović
    The geometrical form for the string space-time action
    *Eur. Phys. J. C 50* (2007) 683-689 (*hep-th/ 0701264*)

# Dejstvo

- Dejstvo

$$S = \kappa \int_\Sigma d^2\xi \sqrt{-g} \left\{ \left[ \frac{1}{2} g^{\alpha\beta} G_{\mu\nu} + \frac{\varepsilon^{\alpha\beta}}{\sqrt{-g}} B_{\mu\nu} \right] \partial_\alpha x^\mu \partial_\beta x^\nu + \Phi R^{(2)} \right\}$$

  - $\xi^\alpha \ (\alpha = 0, 1)$ koordinate svetske površi
  - $x^\mu(\xi) \ (\mu = 0, 1, ..., D-1)$ prostorno vremenske koordinate
  - $x^i(\xi) \ (i = 0, 1, ..., p)$ koordinate Dp-brane
  - Struna propagira u prostoru sa pozadinskim poljima
    - metrički tenzor $G_{\mu\nu}(x)$
    - antisimetrični tenzor $B_{\mu\nu}(x) = -B_{\nu\mu}(x)$
    - dilatonsko polje $\Phi(x)$

- Prostorno vremenske jednačine kretanja (Posledica kvantne konformne invarijantnosti 2-dimenzionog dejstva)

$$\beta^G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{4} B_{\mu\rho\sigma} B_\nu{}^{\rho\sigma} + 2 D_\mu a_\nu = 0$$

$$\beta^B_{\mu\nu} \equiv D_\rho B^\rho{}_{\mu\nu} - 2 a_\rho B^\rho{}_{\mu\nu} = 0$$

$$\beta^\Phi \equiv 2\pi\kappa \frac{D-26}{6} - \frac{1}{24} B_{\mu\rho\sigma} B^{\mu\rho\sigma} - D_\mu a^\mu + 4a^2 = 0$$

$B_{\mu\rho\sigma} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}$ jačina polja
$a_\mu = \partial_\mu \Phi$ gradijent dilatonskog polja

# Kanonska analiza

- Struje

$$J_{\pm\mu} = P^T{}_\mu{}^\nu j_{\pm\nu} + \frac{a_\mu}{2a^2} i_\pm^\Phi = j_{\pm\mu} - \frac{a_\mu}{a^2} j$$

$$i_\pm^F = \frac{a^\mu}{a^2} j_{\pm\mu} - \frac{1}{2a^2} i_\pm^\Phi \pm 2\kappa F' \,, \qquad i_\pm^\Phi = \pi_F \pm 2\kappa \Phi'$$

gde je

$$j_{\pm\mu} = \pi_\mu + 2\kappa \Pi_{\pm\mu\nu} x^{\nu\prime} \,, \qquad \Pi_{\pm\mu\nu} \equiv B_{\mu\nu} \pm \frac{1}{2} G_{\mu\nu}$$

$$j = a^\mu j_{\pm\mu} - \frac{1}{2} i_\pm^\Phi = a^2 (i_\pm^F \mp 2\kappa F')$$

$\pi_\mu$ i $\pi_F$ su kanonski konjugovani impulsi promenljivim $x^\mu$ i $F$

- Kanonski Hamiltonjian i tenzor energije impulsa

$$\mathcal{H}_c = h^- T_- + h^+ T_+$$

$$T_\pm = \mp \frac{1}{4\kappa} \quad G^{\mu\nu} J_{\pm\mu} J_{\pm\nu} + i_\pm^F i_\pm^\Phi \quad + \frac{1}{2} i_\pm^{\Phi\prime}$$

$$= \mp \frac{1}{4\kappa} \quad G^{\mu\nu} j_{\pm\mu} j_{\pm\nu} - \frac{j^2}{a^2} \Bigg) + \frac{1}{2} (i_\pm^{\Phi\prime} - F' i_\pm^\Phi)$$

- Dve nezavisne kopije Virasoro algebre

$$\{T_\pm(\sigma), T_\pm(\bar\sigma)\} = -[T_\pm(\sigma) + T_\pm(\bar\sigma)] \delta'(\sigma - \bar\sigma)$$

# Jednačine kretanja

•

$$[J^\mu] \equiv \nabla_\mp \partial_\pm x^\mu + {}^\star\Gamma^\mu_{\mp\rho\sigma} \partial_\pm x^\rho \partial_\mp x^\sigma = 0$$

$$[h^\pm] \equiv G_{\mu\nu} \partial_\pm x^\mu \partial_\pm x^\nu - 2\nabla_\pm \partial_\pm \Phi = 0$$

$$[i^F] \equiv R^{(2)} + \frac{2}{a^2}(D_{\mp\mu} a_\nu) \partial_\pm x^\nu \partial_\mp x^\mu = 0$$

• Uopštena koneksija

$$^\star\Gamma^\rho_{\pm\nu\mu} = \Gamma^\rho_{\pm\nu\mu} + \frac{a^\rho}{a^2} D_{\pm\mu} a_\nu = \Gamma^\rho_{\nu\mu} \pm P^{T\rho}{}_\sigma B^\sigma_{\nu\mu} + \frac{a^\rho}{a^2} D_\mu a_\nu$$

U odnosu na opše koordinatne transformacije izraz $^\star\Gamma^\rho_{\pm\nu\mu}$ se transformiše kao koneksija.

• Kovarijantni izvod u odnosu na Christoffel-ovu koneksiju $\Gamma^\rho_{\nu\mu}$ i u odnosu na koneksijuand $\Gamma^\rho_{\pm\nu\mu} = \Gamma^\rho_{\nu\mu} \pm B^\rho_{\nu\mu}$, označavamo sa $D_\mu$ i $D_{\pm\mu}$ respektivno

# Koju geometriju vidi probna struna?

- Paralelni transport
  $V^\mu(x) \rightarrow {}^\circ V_\parallel^\mu(x+dx) = V^\mu + {}^\circ\delta V^\mu$

$${}^\circ\delta V^\mu = -{}^\circ\Gamma^\mu_{\rho\sigma} V^\rho dx^\sigma$$

  ${}^\circ\Gamma^\mu_{\rho\sigma}$ afina koneksija

- Kovarijantni izvod

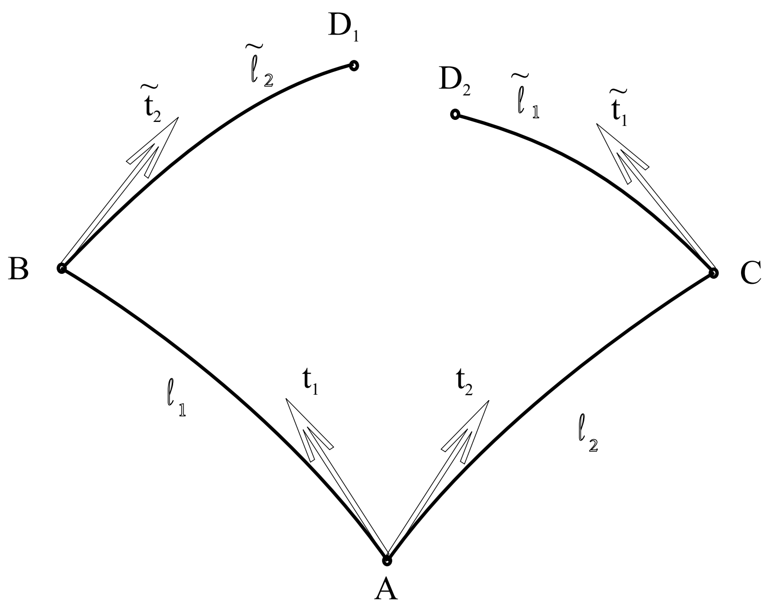
$$\begin{aligned} {}^\circ D V^\mu &= V^\mu(x+dx) - {}^\circ V_\parallel^\mu = dV^\mu - {}^\circ\delta V^\mu \\ &= (\partial_\nu V^\mu + {}^\circ\Gamma^\mu_{\rho\nu} V^\rho) dx^\nu \equiv {}^\circ D_\nu V^\mu dx^\nu \end{aligned}$$

# Torzija

- **torzija** (antisimetrični deo afine koneksije)

$$^{\circ}T^{\rho}_{\mu\nu} = {}^{\circ}\Gamma^{\rho}_{\mu\nu} - {}^{\circ}\Gamma^{\rho}_{\nu\mu}$$

Geometrjska interpretacija: meri ne-zatvaranje zakrivljenog "paralelograma"




$$x^{\mu}(D_2) - x^{\mu}(D_1) = {}^{\circ}T^{\mu}{}_{\rho\sigma}\, t_1^{\rho}\, t_2^{\sigma}\, d\ell_1\, d\ell_2$$

# Ne-metričnost

- Kovarijantni izvod meri razliku vektora u bliskim tačkama

- Koja promenljiva meri razliku dužine vektora ?
  - $V^\mu(x)$: koristeći invarijantnost skalarnog proizvoda u odnosu na paralelni transport
    $V^2(x) = G_{\mu\nu}(x) V^\mu(x) V^\nu(x)$
    $= [G_{\mu\nu}(x) + {}^\circ\delta G_{\mu\nu}(x)] {}^\circ V_\parallel^\mu \, {}^\circ V_\parallel^\nu$
  - i kvadrat njegovog paralelnog transporta ${}^\circ V_\parallel^\mu$ u tačku $x + dx$, ,

$$ {}^\circ V_\parallel^2(x+dx) = G_{\mu\nu}(x+dx) {}^\circ V_\parallel^\mu \, {}^\circ V_\parallel^\nu $$

  - Razlika kvadrata vektora

$$ {}^\circ\delta V^2 = {}^\circ V_\parallel^2(x+dx) - V^2(x) $$

$$ = [G_{\mu\nu}(x+dx) - G_{\mu\nu}(x) - {}^\circ\delta G_{\mu\nu}(x)] \, {}^\circ V_\parallel^\mu \; {}^\circ V_\parallel^\nu $$

S tačnošću do članova višeg reda

$$ {}^\circ\delta V^2 = [dG_{\mu\nu}(x) - {}^\circ\delta G_{\mu\nu}(x)] V^\mu V^\nu $$

$$ = {}^\circ D G_{\mu\nu} V^\mu V^\nu \equiv -dx^\rho \, {}^\circ Q_{\rho\mu\nu} V^\mu V^\nu $$

- **nemetričnost** kovarijantni izvod metričkog tenzora

$$ {}^\circ Q_{\mu\rho\sigma} = -{}^\circ D_\mu G_{\rho\sigma} $$

# Torzija i nemetričnost strune

- **Torzija strune** je antisimetrični deo koneksije strune

$$^\star T_{\pm}{}^{\rho}_{\mu\nu} = {}^\star\Gamma_{\pm}{}^{\rho}_{\mu\nu} - {}^\star\Gamma_{\pm}{}^{\rho}_{\nu\mu} = \pm 2 P^{T\rho}{}_{\sigma} B^{\sigma}_{\mu\nu}$$

- **Nemetričnost strune** meri ne-kompatibilnos metrike $G_{\mu\nu}$ i koneksije strune $^\star\Gamma^{\mu}_{\pm\nu\rho}$

$$^\star Q_{\pm\mu\rho\sigma} \equiv -{}^\star D_{\pm\mu} G_{\rho\sigma} = \frac{1}{a^2} D_{\pm\mu}(a_\rho a_\sigma)$$

Prisustvo dilatonskog polja $\Phi$ dovodi do narušenja prostorno-vremenskog metričkog postulata

- **Weyl vektor strune**

$$^\star q_\mu = \frac{1}{D} G^{\rho\sigma\star} Q_{\pm\mu\rho\sigma} = \frac{-4}{D}\partial_\mu \varphi$$

je gradijent novog skalarnog polja $\varphi$, definisanog izrazom

$$\varphi = -\frac{1}{4}\ln a^2 = -\frac{1}{4}\ln(G^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi)$$

# Prostorno vremensko dejstvo

$$S = \int dx \sqrt{-G} e^{-2\Phi} [R - \frac{1}{12} B^2 + 2D^2\Phi]$$

$B^2 = B_{\mu\nu\rho} B^{\mu\nu\rho} \qquad D^2\Phi = G^{\mu\nu} D_\mu \partial_\nu \Phi$
može se reprodukovati dejstvom

$$^\star S = \int d^D x \ \sqrt{-G} e^{-2\varphi} \ ^\star\mathcal{L}$$

$$^\star\mathcal{L} \equiv {}^\star R + \frac{1}{48} \quad 11\,{}^\star T^2 - 26\,{}^\star Q^2 \quad + \frac{1}{3} \quad \frac{5D}{4} \quad {}^\star q^2$$

Ako je zadovoljen uslov
$\varphi = \Phi \Longleftrightarrow G^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - e^{-4\Phi} = 0$
s tačnošću do člana sa faktorom $\frac{1}{a^2}$ sledi $\qquad S = {}^\star S$

Invarijantna mera

1.Invarijantna u odnosu na prostorno vremenske opšte koordinatne transformacije.

2. Održava se pri paralelnom transportu $\Longleftrightarrow {}^\star D_{\pm\mu} {}^\star\Omega = 0$.

3. Dozvoljava parcijalnu integraciju, koja se može ostvariti uz pomoć Leibniz-ovog pravila i relacije

$$\int d^D x \, {}^\star\Omega \, {}^\star D_{\pm\mu} V^\mu = \int d^D x \partial_\mu ({}^\star\Omega V^\mu)$$

tako da možemo koristiti Stokes-ovu teoremu.