$SU(5/1)$ supergroup and its representation 16 of one generation of fundamental fermions

Djordje Šijački

Institute of Physics, Belgrade, Serbia

It has been suggested that the Standard Model Electro-weak symmetry $SU(2) \times U(1)$ group be embedded in the graded "supergroup" $SU(2/1)$. The successful features relate mostly to the composition of the spectrum of leptons and quarks and to the Higgs field assignment. We consider the $SU(5/1)$ graded algebra that contains the $SU(3)$ symmetry of QCD as well, and construct the **16** dimensional $SU(5/1)$ irreducible representation that accommodates all particles of one generation of basic fermions.

In the Standard Model Electro-weak theory, spontaneous symmetry breakdown is achieved by the "Higgs mechanism" involving a scalar field H, with a λH^2 interaction. The mechanism is triggered by putting in "by hand" a negative squared-mass term $-m^2 H^2$, which breaks the original dilation symmetry required by the Yang-Mills and manifestly broken chiral symmetry. The Weinberg-Salam theory involves a large number of independent assumptions: six $SU(2) \times U(1)$ assignments for quark and lepton chiral field components and for the Higgs Goldstone scalar field ϕ, as well as five couplings $g, \sin^2 \theta_W, -m^2, \lambda, g_H$.

It was suggested independently by Ne'eman and Fairlie that $SU(2) \times U(1)$ be embedded in the minimal graded ("super") group $SU(2/1)$. There are basic 3 and 4 irreducible representations that provide the correct assignment of the lepton and quark $SU(2) \times U(1)$ quantum numbers. Furthermore, the arbitrariness in the Standard Model assignments is additionally reduced by assigning the scalar field ϕ to the odd part of the adjoint representation, with the gauge-field ghosts in the even part according to the $SU(2) \times U(1)$ Yang-Mills structure. The assignment for ϕ is now forced by graded algebra structure. All couplings except for $-m^2$ are related, due to simple $SU(2/1)$ group, and the eleven independent choices are reduced to four.

The original theory encountered two basic difficulties: the apparent loss of the spin-statistics correlation in representations and the fact that $\sin^2\theta_W = .25$ involves normalization of the algebra of $SU(2/1)$ generators (while supertraces yield no result). The answer was given in the form of a method for the gauging of an internal graded group [J. Thierry-Mieg and Y. Ne'eman]. Furthermore, these results were derived from the "superconnection" geometry [Y. Ne'eman, C. Y. Lee and Dj. Šijački]. The theory was "rediscovered" [R. Coquereaux, B. S. Balakrishna, R. Hussling et al.] and linked with the methods of non-commutative geometry [A. Connes and J. Lott]. A detailed presentation is given in a recent review paper [Y. Ne'eman, S. Sternberg, and D. Fairlie, *Phys. Rep.* **406** (2005) 303].

The $SU(5/1)$ algebra

The Gell-Mann-like $SU(N)$ generators λ_{mn}, $m,n=1,2,\ldots,N$ satisfy the following commutation and anticommutation relations

$$[\lambda_{kl},\lambda_{mn}]=if^{op}_{kl\ mn}\lambda_{op},\quad \{\lambda_{kl},\lambda_{mn}\}=ig^{op}_{kl\ mn}\lambda_{op}+\frac{1}{N}\delta_{kl\ mn},$$

where $f^{op}_{kl\ mn}$ are the $SU(N)$ group structure constants, antisymmetric in pairs of lower indices, while $g^{op}_{kl\ mn}$ are the symmetric "g" coefficients.

The $SU(5/1)$ group is defined in a 6-dimensional vector space of 5 even ("bosonic") and 1 odd ("fermionic") dimensions.

The 25 **even** $SU(5/1)$ **generators**:

$SU(3)$ color:

$$X_{ab} = \lambda_{ab}, a \neq b = 1,2,3,\ \ X_{11} = \frac{1}{2} diag\{1,-1,0,0,0,0\},\ \ X_{33} = \frac{1}{\sqrt{6}} diag\{1,1,-2,0,0,0\},$$

$SU(2) \times U(1)$:

$$I_{ij} = \lambda_{3+i\ 3+j},\ \ i \neq j = 1,2,\ \ I_3 = \frac{1}{\sqrt{2}} diag\{0,0,0,1,-1,0\},\ \ Y = \frac{1}{\sqrt{6}} diag\{0,0,0,1,1,2\}$$

$$U = \frac{1}{\sqrt{30}} diag\{1,1,1,1,1,5\},\ \ V_{ai} = \lambda_{ai},\ \ V_{ia} = \lambda_{ia}$$

are supertraceless (trace in the first 5 dimensions minus trace in the 6th dimension) and obey commutation relations.

The remaining 10 $SU(5/1)$ **odd generators**:

$Q_{a6} = \lambda_{a6}$, $S_{i6} = \lambda_{3+i\ 6}$, $Q_{6a} = \lambda_{6a}$, and $S_{6i} = \lambda_{6\ 3+i}$ are supertraceless by definition and they obey anticommutation relations. The $SU(2/1)$ subalgebra of the SU(5/1) algebra ensures all significant results of the Electro-weak part of the Standard Model as described by gauging the graded $SU(2/1)$ symmetry group. The V_{ai} and V_{ia} generators transform as irreducible tensor operators characterized by color 3 and 3^* representation labels, and as weak-isospin dublets. The corresponding gauge potential fields couple leptons and quarks of the same chirality. The Weinberg-Salam theory spontaneous symmetry breaking scalar fields are characterized by quantum numbers given by the $SU(3) \times SU(2/1)$ subgroup labels equal to the labels of the S and Q odd generators.

The $SU(5/1)$ Irreducibe representation 16

The procedure of constructing graded and/or super-algebra irreducible representations is as follows: *(i)* construct *à* la Cartan the irreducible representations of the even subalgebra, as described above, *(ii)* reorganize the odd operators into the "raising" and "lowering" ones, *(iii)* determine the so called "vacuum" states annihilated by the "lowering" odd operators, that characterize each super-representation, and finally, *(iv)* apply to each "vacuum" state all combinations of the "rising" odd operators.

The rising and lowering odd operators of the $SU(5/1)$ algebra are:

$$Q_a^{\pm} = \frac{1}{2}(Q_{a6} \pm Q_{6a}),\ a = 1,2,3, \quad S^{\pm}_{+\frac{1}{2}} = \frac{1}{2}(S_{16} \pm S_{61}), \quad S^{\pm}_{-\frac{1}{2}} = \frac{1}{2}(S_{26} \pm S_{62}).$$

We construct, following the superalgebra representation procedure, the 16-dimensional representation of $SU(5/1)$ particle states:

$$\{(u_{1L}, d_{1L}), (u_{2L}, d_{2L}),\ (u_{3L}, d_{3L}),\ (\nu_L, e_L);\ (u_{1R}, d_{1R}),\ (u_{2R}, d_{2R}),\ (u_{3R}, d_{3R}),\ (\nu_R, e_R)\}$$

The $SU(5/1)$ group generators matrices ore of the form

$$\begin{pmatrix} \mathrm{E} & \mathrm{O} \\ \mathrm{O} & \mathrm{E} \end{pmatrix},$$

where E, O denote the even, odd parts, respectively.

The odd part reads explicitly as follows (nonvanishing elements are denoted by the corresponding generator symbol),

$$O = \begin{pmatrix} S^{+}_{-\frac{1}{2}} & S^{+}_{+\frac{1}{2}} & 0 & 0 & 0 & 0 & Q_1^{+} & 0 \\ S^{-}_{+\frac{1}{2}} & S^{+}_{-\frac{1}{2}} & 0 & 0 & 0 & 0 & 0 & Q_1^{+} \\ 0 & 0 & S^{+}_{-\frac{1}{2}} & S^{+}_{+\frac{1}{2}} & 0 & 0 & Q_2^{+} & 0 \\ 0 & 0 & S^{-}_{+\frac{1}{2}} & S^{+}_{-\frac{1}{2}} & 0 & 0 & 0 & Q_2^{+} \\ 0 & 0 & 0 & 0 & S^{+}_{-\frac{1}{2}} & S^{+}_{+\frac{1}{2}} & Q_3^{+} & 0 \\ 0 & 0 & 0 & 0 & S^{-}_{+\frac{1}{2}} & S^{+}_{-\frac{1}{2}} & 0 & Q_3^{+} \\ 0 & 0 & 0 & 0 & 0 & 0 & S^{+}_{-\frac{1}{2}} & S^{+}_{+\frac{1}{2}} \\ 0 & 0 & 0 & 0 & 0 & 0 & S^{-}_{+\frac{1}{2}} & S^{+}_{-\frac{1}{2}} \end{pmatrix} .$$